DOCUMENT RESUME

ED 260 902 SE 045 937

AUTHOR	DeGuire, Linda J.
TITLE	The Structure of Mathematical Abilities: The View from Factor Analysis.
PUB DATE	Apr 85
NOTE	29p.; Paper presented at the Annual Meeting of the American Educational Research Association (69th, Chicago, IL, March 31-April 4, 1985).
PUB TYPE	Reports - Research/Technical (143) -- Speeches/Conference Papers (150)
EDRS PRICE	MF01/PC02 Plus Postage.
DESCRIPTORS	*Cognitive Ability; Educational Research; *Factor Analysis; *Intelligence; *Mathematics Achievement; *Mathematics Education
IDENTIFIERS	*Mathematics Education Research

ABSTRACT

This study attempted to clarify the structure of mathematical abilities through reanalysis of 48 factor-analytic studies. The factors were compared in six families: General, Numerical, Reasoning, Spatial, Verbal, and Mathematical. Eleven data sets were reanalyzed, with principal factor analysis followed by graphical rotation to oblique, simple structure. Higher-order analyses followed the same procedure; the entire structure was then orthogonalized and the results compared to each other and to the original results. A partial hierarchical structure of mathematical abilities appeared. Overall, reasoning abilities were closely associated with mathematics achievement, while numerical and spatial abilities were associated with only certain aspects of mathematics achievement and verbal abilities were only minimally associated. Evidence was found for a kind of mathematical factor. Among the findings of the reanalyses was that fluid- and crystallized-intelligence abilities were closely associated with mathematics achievement. The variance for each factor was ascertained; collectively, the factors in the reanalyses accounted for about 50% of the variance, most of which represented the relationship of mathematical abilities to general cognitive abilities. Appended is the pool of selected studies and a chart of families of factors. (MNS)

THE STRUCTURE OF MATHEMATICAL ABILITIES:

THE VIEW FROM FACTOR ANALYSIS

by

Linda J. DeGuire

School of Education
University of North Carolina at Greensboro
Greensboro, NC 27412-5001, USA

Paper presented at the Annual Meeting of the American Educational Research Association, Chicago, March 31-April 4, 1985.

ABSTRACT

The study attempted to clarify the structure of mathematical abilities. Forty-eight factor-analytic studies (1938-1979) were selected and their results conceptually synthesized. The factors were compared in six families: General, Numerical, Reasoning, Spatial, Verbal, and Mathematical. Eleven data sets were reanalyzed from: Barakat (1951), Campbell (1956), Kline (1959), Mitchell (1938), Very (1967), Weiss (1955), Werdelin (1958), and Wrigley (1958). In each reanalysis, principal factor analysis was followed by graphical rotation to oblique, simple structure. Higher-order analyses followed the same procedure. The entire structure was orthogonalized by the Schmid-Leiman (1957) procedure. The results were compared to each other and to the original results. Percents of total variance accounted for by factors were also used to examine the association of each family with mathematical abilities.

A partial hierarchical structure of mathematical abilities appeared across the original and reanalyzed results. Overall, reasoning abilities were closely associated with mathematics achievement. Numerical and spatial abilities were associated with only certain aspects of mathematics achievement. Verbal abilities were minimally associated with mathematics achievement. There was evidence for a kind of mathematical factor. In the reanalyses, fluid- and crystallized-intelligence abilities were closely associated with mathematics achievement. Evidence for the automatization of responses was found that extended to certain algebraic skills and revealed sex differences. Some algebraic factors appeared to split according to the cognitive processes involved in the tasks. Factors in the Numerical and Spatial families accounted for about 7% and 5% of the variance in mathematical abilities as measured by the test batteries in the reanalyses. The percents of variance accounted for by the Reasoning, Verbal, or Mathematical families were less definite. Fluid- and crystallized-intelligence-like factors each accounted for approximately 9% of the variance. Higher-order factors together accounted for approximately 25% of the variance, as did first-order factors together. Collectively, the factors in the reanalyses accounted for about 50% of the variance, most of which represented the relationship of mathematical abilities to general cognitive abilities. The diversity among the studies that prevented a statistical synthesis of the results strengthened the generality of the finding concerning structure.

Problem Statement and Background

The role of mathematics in modern society is steadily growing. Its methods and styles of thinking permeate an increasing number of fields. Achievement in mathematics is a critical filter that controls access to many careers, both scientific and nonscientific. However, individual differences in mathematical abilities exist that affect the students' achievement in mathematics. The careful identification and development of each person's mathematical abilities, even though difficult, is important. Improved instruments and strategies for such identification and development depend upon an understanding of the structure of the mathematical abilities they are identifying and developing. The present study was an attempt to clarify our understanding of that structure by examining and reanalyzing factor-analytic studies of mathematical abilities.

Mathematical Abilities--Nature and Structure

There is a considerable literature on mathematical abilities. Recently, the Mathematical Abilities Project at the University of Georgia collected over 1500 references on the topic. These references represent a wide variety of research methods and of approaches to mathematical abilities.

In the literature, there is no agreement on the nature of mathematical abilities. However, most authors have operationally defined, implicitly or explicitly, "mathematical abilities" in terms of the abilities to do mathematics in the school curriculum. Certainly, the abilities to do school mathematics are only a subset of the abilities the research mathematician uses to create mathematics. The learning of scool mathematics, however, is a necessary prerequisite to creating any mathematics that is built upon previous mathematical knowledge. Since most of the mathematical tasks in the studies examined were from the school curriculum and most of the subjects were students, the term mathematical abilities was used to mean the abilities to do school mathematics.

Nor is there agreement in the literature on whether the term "mathematical abilities" should be singular or plural. The apparent singularity or plurality varies with one's (spective. If one is studying abilities in broad areas of human performance or in diverse areas of the school curriculum, the singular term "mathematical ability" fits the perspective. However, if one studies performance in just mathematics or some part of mathematics, then the plural term "mathematical abilities" fits the perspective. Even with the latter perspective, the singularity or plurality of such components as algebraic abilities may vary with the researcher's perception. In the present examination, studies in which the focus is mathematics or some subset of it were considered. Thus, the plural term "mathematical abilities" was used.

Once the plural "mathematical abilities" is accepted, questions arise about the relationships of those abilities among themselves and to other cognitive abilities, that is, of the structure of mathematical abilities. If several measures are chosen to represent mathematical abilities and other cognitive abilities, such that each measure is hypothesized to represent only one ability, the relationships among these measures can be explored through the cluster of statistical techniques known as factor analysis. Since there are a variety of circumstances that may cause measures to associate together on a factor, many of which are irrelevant to the underlying abilities to perform the task or are specific to the sample and the tasks being studied, it is necessary to establish stable factors. Stable factors are factors that have shown up across studies using different populations but the same measures and across studies using different measures but the same population. A collection of stable factors would represent at least part of the structure of mathematical abilities.

Rationale and Questions

The structure of mathematical abilities, as studied primarily through factor analysis, is well represented in the literature on mathematical abilities. Begle (1979) has said, about mathematics education in general:

> There exists, in the literature, a solid body of information about mathematics education. . . . Once this information is dug out and organized, then it will begin to suggest testable theories and at the same time will provide a template against which tentative theories can be tested. (pp. 156-157)

The structure of mathematical abilities is one of the topics to which Begle's strategy should be applied. Though the factor-analytic studies of mathematical abilities had been reviewed (e.g., Krutetskii, 1968/1976; Werdelin, 1958; Wrigley, 1958), there had been no attempt to synthesize the results of the studies. There had been no comprehensive attempt even to compare the factors found among the studies. The present study was designed to organize the information on the structure of mathematical abilities, as seen through factor analysis, in order to suggest testable theories of that structure.

In the present examination, 48 such studies (listed in Appendix A and hereafter referred to as the pool of studies) were selected. It is reasonable to ask what factors have been established as stable. Unfortunately, vast dissimilarities in measures and populations are found in the studies in the pool. Thus, any comprehensive effort to find stable factors would not have been successful. However, it was possible to find some similarities in the factors from different studies. The first purpose of the present study was to synthesize the results in the selected studies about the structure of mathematical abilities.

All the studies in the pool used factor-analytic techniques to study the structure of mathematical abilities. Difference amon the techniques and deficiencies in several of the analyses affected the comparison of results across studies. It seemed likely that reanalysis of the data would yield more precise and, thus, more interpretable and comparable results. Carroll (1980a, p. 121) found a reanalysis of data from studies of cognitive abilities to be useful in clarifying the patterns of results and called for such reanalysis of data from factor-analytic studies of mathematical abilities (1980b, pp. 42-43). The second purpose of the present study was to reanalyze the data from some of the better designed studies in Appendix A in order to clarify the patterns of results.

Another way to clarify these patterns is by means of the comparison of higher-order factors. In the study of intelligence, Tyler (1965, p. 93) concluded that a hierarchical system is the type of theory that best fits the facts. The analysis of higher-order factors is one way to develoo such a hierarchical system. Since then, Cattell's (1971; Hakstian & Cattell, 1978) second-order functions (especially fluid and crystallized intelligence), which result from higher-order analysis, have become widely accepted in the study of human abilities (Horn, 1976; Carroll, 1980c). However, most of the studies of mathematical abilities were completed before attention was given to higher-order analysis. The third purpose of the present study was to make higher-order analyses of the reanalyzed data from some of the better-designed studies in the pool in order to look for a hierarchical structure of mathematical abilities and, if found, to compare it to Cattell's (1971; Hakstian & Cattell, 1978) triadic theory of ability structure.

Procedures

The procedures involved three phases: (1) selecting the pool and making a conceptual synthesis of the original results across the selected studies, (2) selecting the studies for reanalyses and performing the reanalyses, and (3) synthesizing the reanalyzed results.

In the first phase, selection of studies for the pool was based on the criterion that the study used factor analysis to address the question of the structure of mathematical abilities. Originally, five families of factors were formed, with definitions quite similar to definitions of standard factors: General, Numerical, Reasoning, Spatial, and Verbal. Initial work in this framework (DeGuire, 1980) left "a rather long 'laundry list' of factors under 'Other'" (p. 4), and several studies could not be categorized in this scheme. So, the definitions of each family were broadened and the Mathematical family was added. A family of factors represented a class of abilities, possibly broad and divergent but in some way associated with each other. No claim was made that each family represented a single ability or even a small number of closely related abilities. Then, each factor in each study in the pool was classified in one of the families (see Appendix B) and each family was examined for similarities and differences among the factors. The procedures for the examination of each family were neither fixed nor uniform. Perhaps the underlying method could be described as "vote-counting," with the design and significance of the study considered in weighing its vote.

In the second phase, studies were selected for reanalysis primarily on the basis of quality of design, and secondarily on the basis of historical significance, quality of original analysis, and characteristics of the group of studies to be reanalyzed. In each reanalysis, the original correlation matrix was subjected to a principal factor analysis with communality estimates in the diagonal of the matrix, followed by an initial Varimax rotation of the factor matrix. The number of factors to retain was decided by joint consideration of several criteria, including but not limited to discontinuities in the graph of the eigenvalues, Cattell's scree test, and Kaiser-Guttman's criterion. The accepted factor matrix was then graphically rotated to oblique simple structure to maximize primarily the $|.10|$ hyperplanes and secondarily the $|.05|$ hyperplanes. Next, the procedure was repeated with the correlation matrix of the first-order factors to yield one or more second-order factors. If appropriate, the procedure was repeated with the correlation matrix of the second-order factors to yield a third-order factor. The entire structure was then orthogonalized by the Schmid-Leiman (1957) procedure and the orthogonalized factors interpreted.

In the third phase, the reanalyzed results were organized into the same families as in the first phase, and the results were compared to each other and to the results in the original pool. Percents of total variance accounted for by factors in the family were also used to examine the association between factors in the family and mathematical abilities.

Comparison of Original Results across Studies

The 48 studies in the pool (see Appendix A) were published from 1938 through 1979 in 8 different countries. Their samples included thousands of subjects, ranging in age from 7 or 8 years old to adulthood, with the majority being boys, ages 11 or 12 through 17 or 18. In certain studies containing multiple data sets (e.g., Dunkley, 1976; Lee, 1956; Wrigley, 1958) the results from some data sets were pooled and treated as though from a single data set. In all, there were 70 data sets. Each factor for each data set in each study was classified in one of the families and the chart in Appendix B produced. More than 90% of the almost 400 factors in the selected studies (specifically, 345 of 373 factors) could be categorized into one of the families.

The General Family of Factors

The General family of factors consists of two kinds of general factors: the large general factor extracted first in an analysis, and general factors extracted in a second-order analysis. The abilities underlying the General family are those related to fluid and crystallized intelligence. Potentially, the nature and generality of the abilities might vary greatly since membership in the family is based solely on technicalities in the analysis and not on the nature of the factor and measures that load on it. A more heterogeneous pool of studies might not have produced a family with psychological coherence.

The authors of half the studies in the pool extracted a General factor. Ten extracted a large general factor first in the analysis; fourteen extracted one or more general factors in a second-order analysis. Though the two kinds of General factors in the pool were arrived at by quite different methods of analysis, they are relatively similar in the abilities they represent. They commonly represent abilities to discover and use relationships in a variety of tasks and are some combination of fluid and crystallized intelligence. They are quite narrow compared to their potential scope, as illustrated in Cattell's (1971; Hakstian & Cattell, 1978) triadic theory of ability structure. However, the General family is relatively unexplored in the selected studies. So, the results do not imply that other general functions (e.g., Visualization Capacity, General Memory Capacity, or General Retrieval Capacity, in Hakstian & Cattell, 1978) play no role in mathematical abilities. Their role has not been explored in the studies, primarily because of an insufficient base of first-order factors.

Since the General family of factors has been relatively unexplored in the studies, its role in mathematics achievement is also relatively unexplored. What attention was given to this role shows general factors to be closely related to mathematics achievement. Such a relationship is intuitively plausible when one recognizes the emphasis on reasoning in many of the tests used to define general intelligence.

The Numerical Family

The Numerical family of factors consists primarily of the Number factor and secondarily of perceptual speed factors and of computational factors that include reasoning or algebraic skills. It represents speed and facility in manipulating numbers and other symbols according to simple or well-practiced rules. If the rules are well-practiced and originally involved reasoning, the factors are called "numerical." If the rules are simple and originally involved little or no reasoning, the factors are called "perceptual."

Of the 39 selected studies for which a factor was classified in the Numerical family, 37 have one or more numerical factors. Many of the numerical factors in the selected studies have varying degrees of correspondence to the standard Number factor. That is, they are identified by variables measured by highly speeded and simple computational tasks. Yet, there are also important differences among the factors. The greatest source of differences is the presence or absence of significant loadings by reasoning variables. The amount of reasoning decreases with age until, in the studies with older subjects, the numerical factors involve few or no reasoning variables. Werdelin's (1958) automatization process can be used to explain the increasing clarity of the factors. Automatization is a process by which performance on tasks becomes automatic, without conscious effort. So, as facts and skills are practiced more, they become clearer indicators of individual differences in automatization. Further, under certain circumstances, tasks other than the basic operations with whole numbers might become automatized and load solely on the Number factor or define a separate factor. Several investigators found more than one numerical factor; in every case, one factor could be identified with the standard Number factor, and the others involved variables with complex operations and, sometimes, reasoning variables. The presence of reasoning variables suggests that these numerical factors might be characterized by less complete automatization than the Number factor. Also, there is clear evidence in studies in the pool that certain algebraic skills are automatizable. It is not clear to what extent or which ones can be automatized.

Many studies, both within and outside the present pool, deal with the relationship between numerical abilities and mathematics achievement. In general, though numerical abilities initially relate to achievement in school mathematics, their influence declines as the student progresses in the mathematics curriculum. Thus, numerical abilities seem to be basic to mathematics achievement but not integral to it. In other words, a certain minimal level of numerical ability seems essential to achievement in mathematics, but ability beyond this level may be irrelevant to such achievement.

Very few studies in the pool--only five--included variables to define perceptual factors. In these studies, mathematical variables do not load on these factors nor do the factors seem to be related to mathematics achievement.

The Reasoning Family

The Reasoning family consists of a wide variety of factors. It includes such standard factors as Induction, Deduction, General Reasoning, Judgment, and Integration. General Reasoning is referred to as Arithmetic Reasoning here because it is identified by variables that measure the subject's ability to solve arithmetic word problems. Within the studies in the pool, the family also includes factors with such names as Verbal Reasoning, Approach-to-problem-solving, Abstract Reasoning, and Implicit Reasoning. All represent abilities to reach or evaluate conclusions about a situation or a set of data, based on given or assumed information.

In all but nine studies in the pool, the researchers identified at least one reasoning factor, yielding a total of 145 factors, substantially more than in any of the other families. Clearly, the researchers believed reasoning abilities to be correlated with mathematical abilities, and in some cases even equivalent to mathematical abilities. Yet, the same inconsistencies in reasoning factors that are seen in the general factor-analytic literature are also found in the Reasoning family within the pool. The present examination did not satisfactorily simplify the Reasoning family. It served mainly to confirm the close correlation between reasoning and mathematical abilities and did little to clarify the nature and details of the relationship.

Fifty of the reasoning factors have loadings by mathematics achievment variables other than Arithmetic Reasoning variables. The most striking feature of these mathematics achievement variables is their variety and scope. Almost every kind of mathematics variable in the pool loads on some reasoning factor--variables that represent computation, understanding, and routine problem solving in arithmetic, algebra, and geometry. The results of the studies in the pool strongly support a close association between reasoning abilities and mathematics achievement. However, further explorations of the relationship between reasoning and mathematics achievement variables were not fruitful.

The Spatial Family

The Spatial family of factors includes the standard factors of Spatial Orientation, Visualization, Spatial Relations, and Flexibility of Closure. It involves the abilities to perceive, understand, manipulate, and relate spatial material. Most of the spatial factors in the selected studies are defined by variables that identify more than one of the standard factors. One or more spatial factors were identified in 22 studies in the pool, for a total of 35 spatial factors. No two of the spatial factors can be said to be identical, even factors found by the same researcher. And in many of the studies, the spatial factor is merely identified as a spatial factor, with little or no further interpretation, or it is related to a well-known spatial factor.

It was widely believed by the investigators represented in the pool that spatial abilities play a role in mathematics achievement. Some investigators believed the role was so obvious that they reiterated its importance even though the mathematical variables they included did not load on their spatial factor. The evidence from the pool suggests that spatial abilities play a role in mathematics achievement, but only in certain aspects of it. There is no evidence to establish a relationship between spatial abilities and arithmetic achivement or between spatial abilities and algebraic achievement. However, there is evidence to both establish and support a relationship between spatial abilities and geometric achievement. Werdelin's

(1961) results clearly establish a relationship between spatial abilities and certain geometric tasks. Other results within the pool are not inconsistent with his results and, in some cases, support them. However, as several studies suggested, other factors such as general intelligence are more important to geometric achievement than spatial abilities.

The Verbal Family

The Verbal family of factors consists primarily of such factors as Verbal Comprehension, Word Knowledge, and Reading and secondarily of fluency factors. It represents abilities to comprehend language in single words, sentences, or paragraphs (verbal factors) and to produce words with certain restricting conditions (fluency factors). Overall, verbal abilities seem to have minimal influence on mathematics achievement once basic reading abilities have been developed. The influence of fluency factors on mathematics achievement has not been explored.

The Mathematical Family

The Mathematical family consists of factors that represent specific learning in mathematics or some part of mathematics. The factors can be considered agencies in Cattell's (1971) triadic theory of ability structure. The boundaries for this family were difficult to set because they are essentially the distinction between ability and achievement. Many psychometricians have abandoned any attempt to distinguish these concepts. However, the formation of the family provided a framework in which to examine the question of the generality or specificity of mathematical abilities. That is, are all mathematical abilities general abilities applied to mathematics, or are some mathematical abilities specific to mathematics?

In the selected studies, mathematical variables have defined mathematical factors, regardless of the sample or variables in the battery, even though specific mathematical factors have not been identified consistently across studies. Thus, mathematical factors have been clearly established as a kind of factor found in the pool. Does their existence imply the existence of specific mathematical abilities? Clearly, the evidence in the pool shows that mathematics achievement depends upon a variety of factors that are not specific to mathematics. But the existence of mathematical factors also opens up the possibility of specific mathematical abilities. Such abilities are only one possible source that could produce such factors. Personality variables, teacher variables, socio-economic variables, learning style variables, and many others are likely to affect such factors. It is beyond the scope of factor analysis, as used in the studies in the pool, to determine one source out of many for the individual differences that generate a factor. However, even though the mathematical factors do not prove or disprove the existence of specific, mathematical abilities, they do clearly show that not all variance in mathematics achievement is accounted for by general abilities.

Miscellaneous Factors

In twelve studies in the pool, factors were identified that did not fit into any of the families. Most of the miscellaneous factors relate to areas that have been widely explored in the general factor-analytic literature but that have not been explored widely in the selected studies. It would be inappropriate to conclude that they are unrelated to mathematical abilities. Rather, the relationships have not been adequately explored.

The Reanalyses

Twelve data sets from the studies in the pool were chosen for reanalysis. They were: Barakat's (1951) female and male data sets, Campbell's (1956) data set, Canisia's (1962) data set, Kline's (1959) data set B, Mitchell's (1938) data set, Werdelin's (1958) Alpha data set, two of Wrigley's (1956, 1958) data sets, and Very's (1967) female and male data sets. Details of each reanalysis are presented in DeGuire (1983a, pp. 121-180, 247-314). The Canisia reanalysis was terminated due to technical problems in the extraction of first-order factors.

The number of students in the samples for the reanalyses totaled more than 2100, approximately 1700 male and 400 female. The majority of the students were 13, 14, or 15 years of age but one sample (Campbell's) consisted of sixth-grade boys and two samples (Very's) consisted of college students. The samples were from four countries: United States, England, Northern Ireland, and Sweden.

Comparison of Results across Reanalyses

The General Family

In the reanalyses, more, and more clearly defined, second-order factors were extracted than in the pool. Also, third-order factors were extracted in the reanalyses, though most of them were vague. Cattell's (1971) higher-stratum functions in his triadic theory of ability structure were used as a basis for comparing the higher-order factors in the reanalyses. As would be expected from the kinds of tests included in the batteries of the reanalyses, most identifications of higher-order factors were made with fluid intelligence (Gf) and crystallized intelligence (Gc). The other higher-stratum functions are too underrepresented to suggest any patterns. Further discussion will be limited to fluid and crystallized intelligence.

Nine second-order factors in the reanalyses could be roughly identified with fluid intelligence (Gf). The percents of total variance accounted for by the factor had a mean of 8.5% with a standard deviation of 3.4%. Only five of the nine percents of total variance fell within one standard deviation of the mean. The loadings of mathematical first-order factors on second-order Gf-like factors suggested that mathematics achievement is significantly associated with indi idual differences in fluid intelligence. Of the fourteen mathematical factors in the reanalyses, only one mathematical factor did not load significantly (i.e., /.30/ or higher) or weakly (i.e., /.20/ to /.30/) on a Gf-like factor when such a factor was identified, nine loaded significantly on Gf-like factors, and four were in reanalyses (Campbell, Kline) in which no Gf-like factor was extracted.

Eight second-order factors in the reanalyses could be roughly identified with crystallized intelligence (Gc). The percents of total variance accounted for by the factors had a mean of 9.2%, with a standard deviation of 5.6%. Only four of the eight percents of total variance fell within one standard deviation of the mean. The loadings of mathematical first-order factors on second-order Gc-like factors suggested that mathematics achievement may be significantly associated with crystallized intelligence. Of the fourteen mathematical factors in the reanalyses, two mathematical factors did not load significantly or weakly on a Gc-like factor when such a factor was identified, nine loaded significantly on Gc-like factors, and three were in reanalyses (Campbell, Wrigley A, Wrigley B) in which no Gc-like factor was extracted.

The Numerical Family

Most of the factors in the Numerical family in the reanalyses were closely related to the standard Number factor, the standard Perceptual Speed factor, or both. That is, they were defined by highly-speeded tests with simple computational tasks for the Number factor or highly-speeded tests with simple perceptual tasks for the Perceptual Speed factor. The percents of total variance accounted for by the Numerical family had a mean of 6.6% and a standard deviation of 3.2%. In all but two of the reanalyses, this percent falls within one standard deviation of the mean.

The loadings of mathematical measures on factors in the Numerical family followed patterns similar to those found in the synthesis of the pool, but also suggested certain sex differences in factors in the Numerical family. In the pool, the loadings of complex computation measures and Arithmetic Reasoning measures on the Number factor seemed to be related to the age of the subjects. In the reanalyses, though, the loadings of complex computation measures also seemed to be related to the sex of the subjects. In those reanalyses in which the sample was all or part female (Barakat Female, Weiss, Very Female), complex computation measures loaded on the Number factor. Another sex difference found in the reanalyses is the loadings of algebraic measures on the Number factor in two reanalyses. In the Barakat Female and Very Female reanalyses, the algebraic tests measure general achievement in elementary algebra and are not highly speeded. Their loadings on the Number factor suggest that, for females, individual differences in the automatization (Werdelin, 1958) represented by the Numerical family are associated with performance in algebra. The same tests did not load on the Number factor in the corresponding Male reanalyses.

The Reasoning Family

In all but one of the reanalyses, factors were identified that have been classified in the Reasoning family. The most common kind of reasoning factor was Inductive Reasoning (I). The percents of total variance accounted for by the Reasoning family had a mean of 6.9% and a standard deviation of 5.0%. In only five of the reanalyses did this percent fall within one standard deviation of the mean.

The loadings of mathematical tests on the factors in the Reasoning family did not clarify the relation between reasoning and mathematical abilities. Numerical measures, complex computational measures, and algebraic measures loaded on some reasoning factor in most of the reanalyses in which they were included; the exceptions were Mitchell and Barakat Female. They did not, however, load consistently with one aspect of general reasoning. No geometric measure loaded on a reasoning factor in the reanalyses, even though such tests were included in the batteries of the Mitchell, Barakat Female, Barakat Male, Weiss, Wrigley-A, and Wrigley-B reanalyses. The geometric measures all loaded on Mathematical factors. The absence of geometric measures may suggest that individual differences in general and arithmetic reasoning play less of a role in geometry achievement than individual differences in the geometry learning environment.

The Spatial Family

In all but one of the reanalyses, factors were identified that have been classified in the Spatial family. In the reanalyses, the factors in the Spatial family tended to be more clearly defined than those in the entire pool. That is, the fraction of the factors dominated by one aspect (spatial orientation or visualization) was larger in the reanalyses than in the entire pool. Most of the factors exhibited more than one aspect, however, and about half included reasoning aspects. The percents of total variance accounted for by the Spatial family had a mean of 4.8% and a standard deviation of 2.2%. In all but two of the reanalyses, this percent fell within one standard deviation of the mean.

The loadings of mathematical tests on spatial factors in the reanalyzed results follow patterns similar to those for the entire pool. Geometric measures loaded significantly, weakly, or appreciably on spatial factors in four of the seven reanalyses in which both kinds of tests were included in the battery. This ratio is larger than in the entire pool. Also, as in the original analysis, no geometric tests loaded on the one factor (WsF) which seemed almost purely Visualization. These results support the conclusion in the synthesis of the original results that certain spatial abilities are associated with the performance of certain geometric tasks.

The Verbal Family

In only eight of the reanalyses were factors identified that have been classified in the Verbal family. The percents of total variance accounted for by the Verbal Family had a mean of 5% and a standard deviation of 3.6%. In only four of the eleven reanalyses did this percent fall within one standard deviation of the mean. Few mathematical tests loaded on the factors in the Verbal family. Those that did were mathematical vocabulary or reading tests or tests with verbally-presented tasks.

The Mathematical Family

In ten reanalyses, factors were identified that have been classified in the Mathematical family. Most of the Mathematical factors in the reanalyses closely paralleled factors in the original results. Consequently, they led to the same conclusions as for the Mathematical family in the entire pool, that is, that the relationship of achievement between and within branches of mathematics clearly depends upon the nature of the measures included in the analysis. Further, the results of the reanalyses strongly supported Mathematical factors as a kind of factor, though they did not establish any particular Mathematical factor as stable.

The Mathematical factors in the reanalyses that did not parallel factors in the original results suggested some interesting observations. The Mathematical Expression Knowledge factor in the Weiss reanalysis seemed to be a mathematical equivalent to a Word Knowledge factor in general vocabulary. Apparently, not all the abilities utilized in mathematical vocabulary tests are the same as abilities utilized in general vocabulary tests. Differences in these abilities have not been identified or explored in the studies in the pool. Mathematical factors in the Kline reanalysis exhibited an interesting distinction among mathematical factors within the same branch of mathematics. They were distinguished by the level of understanding required rather than by the topics within the tasks. Other studies of a single branch of mathematics have identified very specific and content-distinguished factors. Further evidence for an association by level of understanding might have implications for teaching strategies. However, it should also be noted that this distinction was not stable enough across Kline's (1959) A and B samples to be evident in the congruent factors of the original results.

Scope of Abilities Represented

Now that each family of factors has been examined, an important question remains. Do these families and the factors in them represent the entire domain of mathematical abilities? That question is easily answered no. What part or how large a part of mathematical abilities do they represent? In general, they represent the influence of general cognitive abilities on mathematics achievement. The exceptions are the factors in the Mathematical family. The total percents of variance accounted for by each order are given in Table 1. Those percents that do not fall within one standard deviation of the mean for that group are starred. For the second-order, higher-order total, and first-order total groups, almost all of the percents fall within one standard deviation of the mean. These results suggest that the group of first-order factors and the group of higher-order factors each account for approximately one-fourth of the variance in mathematics achievement, as measured bv the tests in the reanalyses. Within the group of higher-order factors, second-order factors account for most of the variance.

Table 1

Percent of Total Variance Accounted for by Orders of Factors for the Reanalyzed Results

Study	Third order	Second order	Higher-order total	First order total	Total
Mitchell	6	15	21	28	49
Barakat					
Female	-	28	28	17*	45*
Male	-	25	25	23	48
Weiss	10	8	18	33	51
Campbell	-	54*	54	12*	66*
Kline B	8	25	33	23	56
Wrigley					
A	3*	20	23	33	56
B	6	17	23	30	53
Werdelin Alpha	18*	9	27	31	58
Very					
Female	6	17	23	32	55
Male	18*	14	32	35	67*
n	9	11	11	11	11
Means	9.4	21.1	27.9	27	54.9
Standard Deviations	5.3	12.1	9.3	7.0	6.6

*More than one standard deviation from the mean.

Hierarchical Structure

If families of factors rather than individual factors are used as the basis for a structure, then at least a partial hierarchical structure of mathematical abilities appears similar across the original studies and the reanalyses. The same diversity among the studies that prevented a statistical synthesis of the resu'ts strengthens the generalizability of the finding concerning structure. Overall, the results suggest the conclusions that reasoning abilities are closely associated with mathematics achievement, that numerical and spatial abilities are associated with certain aspects of mathematics achievement, and that verbal abilities are not closely associated with mathematics achievement. Also, a kind of mathematical factor clearly exists. Further, the reanalyses suggest that the abilities represented by both fluid and crystallized intelligence are closely associated with mathematics achievement.

Factors in the Numerical family appear to account for about 7% of the variance in mathematical abilities, and factors in the Spatial family account for about 5% of the variance. The percents of variance accounted for by the Reasoning, Verbal, or Mathematical families are less definite. Fluid- and crystallized-intelligence-like factors each appear to account for approximately 9% of the variance in mathematical abilities, and the abilities they represent are significantly associated with mathematics achievement. In the reanalyses, the group of higher-order factors and the group of first-order factors each accounted for approximately 25% of the variance in mathematical abilities as measured by the variables in the reanalyses.

Overall, the percent of variance accounted for by the hierarchical factor structures in the reanalyses hovered around 50% (see Table 1). Given the diversity of samples and test batteries in the reanalyses, this similarity is interesting but its meaning is not clear. Possibly it is a ceiling effect for studies of the kind in this examination. Certainly, some fraction of the remaining variance can be attributed to error of measurement and factors irrelevant to mathematics achievement. Test reliabilities in the batteries of the reanalyses were generally .80 to .90 or higher, indicating that 64 to 81% or more of the variance could be accounted for by the factor structures. So a fraction of the variance is still unaccounted for. One can speculate about what kind of factors might account for this remaining fraction--specifically mathematical factors, other cognitive factors, personality factors, or factors irrelevant to mathematics achievement. Probably, it is a combination of these kinds of factors. The result still leaves open the possibility that non-achievement abilities exist that are specific to mathematics.

Though the results in the pool and the reanalyses do not provide a complete st ucture of mathematical abilities, what is known of the structure clearly fits Cattell's triadic theory of ability structure. Most of the results in the pool relate the first level (the agencies) of the triadic theory to mathematical abilities. The results of the reanalyses relate the first and third levels (the agencies and the capacities) to mathematical abilities. Figure 1 shows the mapping between general abilities and mathematical abilities suggested by the results. The small squares within the set of mathematical abilities symbolize the many aspects of mathematical abilities, with those squares inside the small oval representing the abilities examined in this study. The mapping shows the association of spatial, numerical, and verbal abilities with only certain mathematical tasks, the association of reasoning abilities and fluid and crystallized intelligence with many mathematical tasks, and the unexplored association between other capacities and mathematical tasks and between other agencies and mathematical tasks. The small squares within the set of mathematical abilities but outside the small oval are included to leave open the possibility that mathematical abilities are not merely a subset of general cognitive abilities. The partial structure of mathematical abilities that emerged in this study primarily represents a relationship between mathematical abilities and general cognitive abilities. Few of the results in the selected studies offered insights into the <u>structure</u> of abilities specific to mathematics, though their existence was confirmed. When a clearer picture of the structure of mathematical abilities is incorporated into the presently known structure, the relationship of the entire structure to Cattell's triadic theory may be seen to be one of intersecting sets, rather than subset and set.

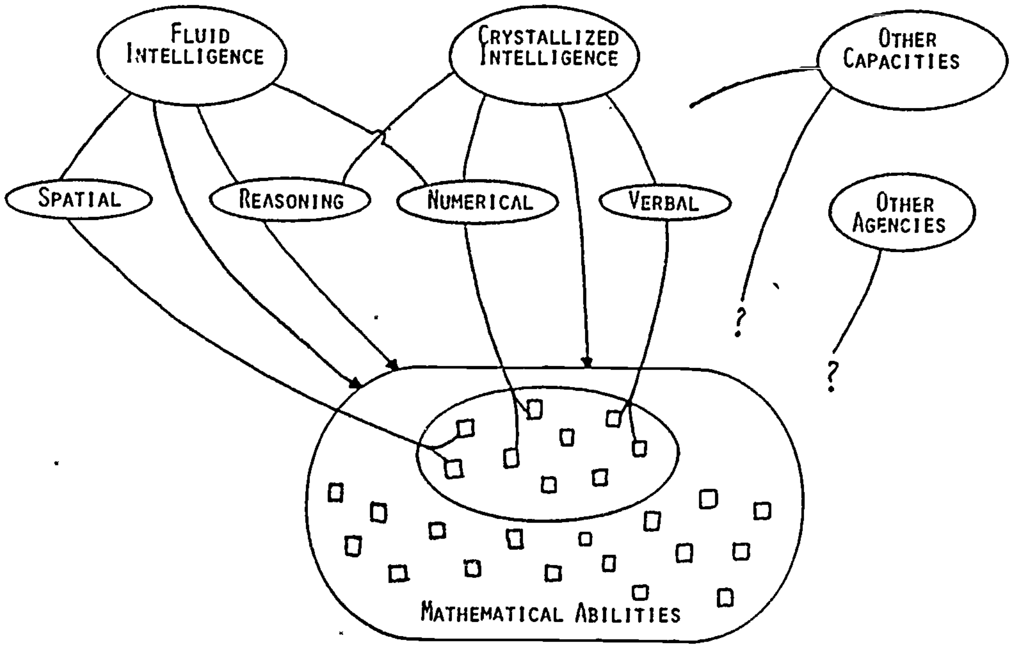

Figure 1. Mapping between General Abilities and Mathematical Abilities

Directions for Future Research

Future research on the structure of mathematical abilities should take into account the results of the present study. Also, the results of the present study need to be complemented by extending the structure of mathematical abilities. What is known of that structure now is primarily the relationship of mathematical abilities to certain general cognitive abilities. However, the relationships of mathematical abilities among themselves are almost totally unexplored in existing studies. Though the results of some studies in the pool suggest that content divisions may serve as hypotheses for relationships among mathematical abilities, the results of the Kline-B reanalysis suggest that levels of understanding may provide hypotheses. If so, the results of information-processing studies and developmental studies are likely to be helpful guides in the exploration. Specifically, the work of Krutetskii (1968/1976) should be given attention. Further, not enough may yet be known about abilities specific to mathematics to allow the field to be studied effectively with factor-analytic designs. The structure of such abilities cannot be studied until the abilities are identified.

Future research might also vary the definition of mathematical abilities. In the pool, mathematical abilities were operationally defined implicitly or explicitly in terms of the abilities to do school mathematics. Such a definition has led to an emphasis on finding relationships between mathematical abilities and those general cognitive abilities that might be useful in predicting further mathematics achievement. One can, however, define mathematical abilities in terms of the abilities to learn rather than to do school mathematics. Such a definition may lead one to study abilities specific to mathematics. Mathematical abilities can also be defined in other ways, each of which would lead one to study certain subsets of abilities. Perhaps one of the most interesting definitions would refer to abilities to create mathematics. When such expansions of the definition of mathematical abilities are considered, one sees that only a small part of mathematical abilities has been investigated.

REFERENCES

Note: References for studies in the pool are given in Appendix A and are not repeated here.

Begle, E. G. Critical variables in mathematics education. Washington DC: Mathematical Association of America and National Council of Teachers of Mathematics, 1979.

Carroll, J. B. Individual difference relations to psychometric and experimental cognitive tasks. Chapel Hill: University of North Carolina, L. L. Thurstone Psychometric Laboratory, Report No. 163, April 1980. (a) [Document AD-A086 057, National Technical Information Service]

Carroll, J. B. Factor-analytic and cognitive approaches to the study of mathematical abilities. Unpublished manuscript, October 1980. (b)

Cattell, R. B. Abilities: Their structure, growth and action. Boston: Houghton-Mifflin, 1971.

DeGuire, L. J. A review of the factor-analytic literature on mathematical abilities. Paper presented at the Fourth International Congress on Mathematics Education, Berkeley, CA, August 1980.

DeGuire, L. J. Mathematical abilities: The View from factor analysis. In S. Wagner (ed.), Proceedings of the Fourth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Athens, Georgia, October 1982.

DeGuire, L. J. Reanalyses of factor-analytic studies of mathematical abilities (Doctoral dissertation, University of Georgia, 1983). Dissertation Abstracts International, 1983, 44, 415A. (University Microfilms, No. DA8314713) (a)

DeGuire, L. J. Reanalyses of factor-analytic studies of mathematical abilities. In J. C. Bergeron & N. Herscovics (eds.), Proceedings of the Fifth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Montreal, September 1983. (b)

Hakstian, A. R., & Cattell, R. B. Higher-stratum ability structures on the basis of twenty primary abilities. Journal of Educational Psychology, 1978, 70, 657-669.

Horn, J. L. Human abilities: A review of research and theory in the early 1970's. Annual Review of Psychology, 1976, 27, 437-485

Krutetskii, V. A. The psychology of mathematical abilities in schoolchildren. (J. Kilpatrick & I. Wirszup, Eds.; J. Teller, Trans.). Chicago: University of Chicago Press, 1976. (Originally published, 1968).

Schmid, J., & Leiman, J. M. The development of hierarchical factor solutions. Psychometrika, 1957, 22, 53-61.

Tyler, L. E. The psychology of human differences (3rd ed). New York: Appleton-Century-Crofts, 1965.

APPENDIX A

THE POOL OF SELECTED STUDIES

Note: When a study has been reported in more than one place, the most accessible report is listed first, with other reports listed immediately below it and indented.

1. Barakat, M. K. A factorial study of mathematical abilities. British Journal of Psychology, Statistical Section, 1951, 4, 137-156. (a)

   Barakat, M. K. Factors underlying mathematical abilities. Unpublished doctoral dissertation, University of London, 1950.

   Barakat, M. K. Factors underlying the mathematical abilities of grammar school pupils. British Journal of Educational Psychology, 1951, 21, 239-240. (b)

2. Blackwell, A. M. A comparative investigation into the factors involved in mathematical ability of boys and girls. British Journal of Educational Psychology, 1940, 10, 143-153, 212-222.

3. Buddeke, R. Differential factorial patterns of boys and girls in algebraic computation (Catholic University of America Educational Research Monograph Vol. 23, No. 1). Washington, DC: Catholic University of America Press, 1960.

4. Campbell, D. F. Factorial comparison of arithmetic performance of boys in sixth and seventh grade (Catholic University of America Educational Research Monograph Vol. 20, No. 2). Washington, DC: Catholic University of America Press, 1956.

5. Canisia, Sr. M. Mathematical ability as related to reasoning and use of symbols. Educational and Psychological Measurement, 1962, 22, 105-127.

Majewska, M. C. A study of mathematical ability as related to reasoning and use of symbols (Pub. No. 12). Chicago: Loyola University, Psychometric Laboratory, February 1960. (a)

Majewska, Sr. M. Canisia. A study of mathematical ability as related to reasoning and use of symbols. Unpublished doctoral dissertation, Loyola University, 1960. (b)

6. Davis, F. B. The mental skills in arithmetic reasoning and their measurement in Air Force selection and classification programs (Project 7719). Lackland Air Force Base, TX: Personnel Laboratory, Wright Air Development Division, Air Research and Development Command, 1961.

7. Donahue, R. T. An investigation of the factor pattern involved in arithmetic problem solving of eighth grade girls (Doctoral dissertation, Catholic University of America, 1969). Dissertation Abstracts International, 1969, 30, 2372A. (University Microfilms No. 69-19, 720)

8. Donohue, J. C. Factorial comparison of arithmetic problem-solving ability of boys and girls in seventh grade (Catholic University of America Educational Research Monograph Vol. 20, No. 5). Washington, DC: Catholic University of America Press, 1957.

9. Dunkley, M. E. A factorial study of the nature of mathematical ability of children in grade four through grade eight. Unpublished doctoral dissertation, Macquarie University, 1976.

10. Dye, N. W., & Very, P. S. Growth changes in factorial structure by age and sex. Genetic Psychology Monographs, 1968, 78, 55-88.

11. Edwards, R. M. Factorial comparison of arithmetic performance of girls and boys in the sixth grade (Catholic University of America Educational Research Monograph Vol. 20, No. 7). Washington, DC: Catholic University of America Press, 1957.

12. Emm, M. E. A factorial study of problem-solving ability of fifth-grade boys (Catholic University of America Educational Research Monograph Vol. 22, No. 1). Washington, DC: Catholic University of America Press, 1959.

13. Furneaux, W. D., & Rees, R. The dimensions of mathematical difficulties (Occasional Publications Series, No. 1). Uxbridge: Brunel University, Department of Education, September 1976.

14. Furneaux, W. D. & Rees, R. The structure of mathematical ability. British Journal of Psychology, 1978, 69, 507-512.

15. Hamza, M. A study of certain aspects of retardation in mathematics amongst secondary grammar school pupils by factorial and individual methods. Unpublished doctoral dissertation, Leeds University, 1951.

Hamza, M. Retardation in mathematics amongst grammar school pupils. British Journal of Educational Psychology, 1952, 22, 189-195.

16. Heidig. Data for arithmetic problem solving of seventh-grade girls. Reported in J. C. Donohue, 1957.

17. Kennedy, W. A., & Walsh, J. A factor analysis of mathematical giftedness. Psychological Reports, 1965, 17, 115-119.

18. Kline, W. E. A synthesis of two factor analyses of intermediate algebra. Psychometrika, 1959, 24, 343-359.

Kline, W. E. A synthesis of two factor analyses of intermediate algebra. Princeton, NJ: Princeton University and Educational Testing Service, June 1956.

19. Lee, D. M. A study of specific ability and attainment in mathematics. British Journal of Educational Psychology, 1956, 26, 178-189.

20. Leton, D. A., & Kim, S. Analysis of mathematical abilities required for success in ninth-grade mathematics. Honolulu: University of Hawaii, Education Research and Development Center, December 1966.

21. Malinen, P. The learning of elementary algebra (Research Bulletin No. 25). Helsinki: University of Helsinki, Institute of Education, January 1969. (Available from Department of Education, University of Helsinki, Fabianinkatu 28, SF-00100 Helsinki 10, Finland).

22. McAllister, B. Arithmetical concepts and the ability to do arithmetic. British Journal of Educational Psychology, 1951, 21, 155-156.

McAllister, B. Arithmetical concepts and the ability to do arithmetic. Unpublished thesis, Glasgow University, 1950.

23. McCallum, D. I., Smith, I. M., & Eliot, J. Further investigations of components of mathematical ability. Psychological Reports, 1979, 44, 1127-1133.

24. McTaggert, H. P. A factorial study of the problem-solving ability of fifth-grade girls. Washington, DC: Catholic University of America Press, 1959.

25. Meyer, R. A. Mathematical problem-solving performance and intellectual abilities of fourth-grade children. Journal for Research in Mathematics Education, 1978, 9, 334-348.

Meyer, R. A. A study of the relationship of mathematical problem solving performance and intellectual abilities of fourth-grade children (Tech. Rep. 379). Madison: University of Wisconsin, Wisconsin Research and Development Center for Cognitive Learning, February 1976.

26. Mitchell, F. W. The nature of mathematical thinking. Educational Research Series (No. 53). Melbourne: Melbourne University Press, 1938.

27. Novello, R. R. Differential factors in algebraic computation for high-achieving boys and girls (Catholic University of America Educational Research Monograph Vol. 23, No. 2). Washington, DC: Catholic University of America Press, 1960.

28-30. Olckers, P. J. A factorial study of arithmetical ability. Journal for Social Research, Pretoria, 1951, 2, 1-21. (3 studies in 1 report.)

31. Pruzek, R. M., & Coffman, W. E. A factor analysis of the mathematical sections of the Scholastic Aptitude Test (ETS RB 66-12). Princeton, NJ: Educational Testing Service, 1966.

32. Rusch, C. A. An analysis of arithmetic achievement in grades four, six, and eight (Doctoral dissertation, University of Wisconsin, 1957). Dissertation Abstracts, 1957, 17, 2217. (University Microfilms No. 57-3568)

33. Sutherland, J. An investigation into some aspects of problem solving in arithmetic. British Journal of Educational Psychology, 1941, 11, 215-222; 1942, 12, 35-46.

34. Symons. Data for arithmetic problem solving of eighth-grade boys. Reported in R. T. Donahue, 1969.

35. Very, P. S. Differential factor structures in mathematical ability. Genetic Psychology Monographs, 1967, 75, 169-207.

Very, P. S. Quantitative, verbal and reasoning factors in mathematical ability (Doctoral dissertation, Pennsylvania State University, 1963). Dissertation Abstracts, 1964, 25, 1371. (University Microfilms, No. 64-7748)

36. Very, P. S., & Iacono, C. H. Differential factor structure of seventh-grade students. Journal of Genetic Psychology, 1970, 117, 239-251.

37. Watters, L. J. Factors in achievement in mathematics: A study of the factor patterns resulting from analyses of the scores of boys and girls in junior year of high school on one mathematics test (Catholic University of America Educational Research Monograph Vol. 18, No. 3). Washington, DC: Catholic University of America Press, 1954.

38. Weber, H. An investigation into the factorial structure of numerical tasks. Psychological Abstract, 1954, 28, 674.

Weber, H. [Investigation of the factor structure of numerical problems]. Zeitschrift fur experimentelle und angewandte Psychologie, 1953, 1, 336-393. (part)

39. Weiss, E. S. A factor analysis of mathematical ability. Unpublished doctoral dissertation, Harvard University, 1955.

40-43. Werdelin, I. The mathematical ability: Experimental and factorial studies. Lund: Gleerup, 1958. (4 studies in 1 report.)

44. Werdelin, I. Geometrical ability and the space factors in boys and girls. Lund: Gleerup, 1961.

45. Werdelin, I. A study of age differences in factorial structure (Didakometry, No. 6). Malmo, Sweden: School of Education, Department of Educational and Psychological Research, February 1966. (a)

46. Werdelin, I. A synthesis of two factor analyses of problem solving in mathematics (Didakometry, No. 8). Malmo, Sweden: School of Education, Department of Educational and Psychological Research, March 1966. (b)

47. Wooldridge, E. T. Factorial study of changes in ability patterns of students in college algebra (Doctoral dissertation, University of Nebraska, 1964). Dissertation Abstracts, 1964, 25, 2867. (University Microfilm No. 64-11, 945)

48. Wrigley, J. The factorial nature of ability in elementary mathematics. British Journal of Educational Psychology, 1958, 28, 61-78.

Wrigley, J. The factorial nature of ability in elementary mathematics. Unpublished doctoral dissertation, Queen's University, Belfast, 1956.

APPENDIX B

CHART OF FAMILIES OF FACTORS

This appendix contains a chart that shows, for each sample in the studies in the pool, the families into which the factors in the study were classified. The column headings are abbreviations for the families, as follows:

G = General family,
N = Numerical family,
R = Reasoning family,
S = Spatial family
V = Verbal family,
M = Mathematical family,
and Misc = miscellaneous factors.

A letter entry in a column indicates a factor was extracted that has been classified in that family. A number preceding the letter indicates the number of factors that have been classified in that family; no number indicates one factor. Two letters separated by a comma (e.g., n,p) indicates two separate factors. Two or more letters not separated by a comma (e.g., nr) indicates one factor that has been classified in more than one family. The letters represent the following:

g(1) = general factor on first order,
g(2) = general factor on second order,
n = numerical factor,
p = perceptual factor,
r = reasoning factor,
s = spatial factor,
v = verbal factor,
f = fluency factor,
m = mathematical factor,
x = miscellaneous factor,

Study Sample	G	N	R	S	V	M	Misc
Barakat, 1951a							
Boys	g(1)	n	vr	s	vr	m	
Girls	g(1)	n	vr	s	vr	m	
Blackwell, 1940							
Boys	g(1)		sr,vr	sr	vr		
Girls	g(1)		sr,vr,r	sr	vr		
Buddeke, 1960							
Boys						5m	
Girls						5m	
Campbell, 1956	g(2)	nr	nr,r		v		
Canisia, 1962	4g(2)	2n	sr,4r	sr	v,f	m	2x
Davis, 1961		2n	4r,sr	sr	v	m	
Donahue, 1969		nr	nr,r		v		
Donohue, 1957	g(2)	nr	nr,r		v		
Dunkley, 1976							
4th grade		2n	mr			mr	
5th grade		n	mr,r			mr	
6th grade		n	mr			mr	
7th grade		3n				m	
8th grade		2n				m	
Dye & Very, 1968							
9th grade male		np	vr,6r		vr		
9th grade female		np	vr,3r		vr		
11th grade male		n,p	vr,6r		vr		
11th grade female		n,p	vr,5r		vr		
college male		n,p	vr,4r		vr	m	
college female		n,p	vr,3r		vr		
Edwards, 1957	g(2)	nr	nr		v		
Emm, 1959	g(2)	nr	sr,nr	sr	v		
Furneaux & Rees, 1976,all samples	g(1)					m	
Furneaux & Rees, 1978	g(1)					m	
Hamza, 1951							
Normal	g(1)	nmr	nmr,sr,r	sr		nmr	
Retarded	g(1)	nr	nr,r				x

Study Sample	Families G	N	R	S	V	M	Misc
Heidig, 1957	g(2)	n,nr	nr,r				
Kennedy & Walsh, 1965		p	r			m	2x
Kline, 1959 Congruent factors		2n	sr	sr	v		x
Lee, 1956 all samples			3r				
Leton & Kim, 1966		n,nr	nr,3r	sr		m	
Malinen, 1969							
grade 7		n	sr,2r	sr			2x
grade 9		n	sr,mr	sr		m,mr	2x
McAllister, 1951	g(1)	nr	nr				
McCallum, et al. 1979, total	g(1)			s	v		2x
McTaggert, 1959	g(2)	nr	nr,r		v		
Meyer, 1978		p	3r		vf	m	
Mitchell, 1938	g(1)		sr	sr	v		x
Novello, 1960							
Boys						5m	
Girls						5m	
Olckers I, 1951		2n	r				x
Olckers II, 1951		2n	r				x
Olckers III, 1951		2n	r	s	vf		x
Pruzek & Coffman 1966	g(2)	n	r			m	3x
Rusch, 1957 all samples		nr	nr			2m	
Sutherland, 1942	g(1)	nr	vr,nr,r		vr		x
Symons, 1969		nr	nr,r		v		

Study Sample	Families G	N	R	S	V	M	Misc
Very, 1967							
Total		n,p	mr,5r	s	v	mr	x
Male		n,p	sr,mr,6r	s,sr	v	mr	x
Female		nr,p	nr,5r	s	v		
Very & Iacono, 1970							
Males		np	4r	s	v		
Females		np	2r	s	v		
Watters, 1954							
Boys	g(2)					3m	
Girls	2g(2)					4m	
Weber, 1954							
Boys		n			f		3x
Girls		n			f		3x
Weiss, 1955	g(2)	n	sr,2r	s,sr	v	3m	
Werdelin, 1958							
Alpha Study	g(2)	n	r,mr	s	v	mr	
Werdelin, 1958							
Sub A Study	g(2)	n	r,mr	s	v	mr	
Werdelin, 1958							
Sub B Study	g(2)	n	r	s	v		
Werdelin, 1958							
Beta Study	g(2)	2n	2r,mr	s	v	mr	
Werdelin, 1961							
Boys A		n	mr	2s	v	mr	
Boys B		n	mr	2s	v	mr	
Girls		n	mr	s	v	mr	
Werdelin, 1966a							
Congruent factors		n	r	s	v		
Werdelin, 1966b							
Synthesis		n	r,mr	s	v	mr	
Wooldridge, 1964							
Beginning		n			vf	2m	
End		n			vf	2m	
Wrigley, 1958							
Combined group	g(1)	n		s	v	m	